1- Publications in Ship Structural Analysis and Design (1969-2002)

1- " Effect of Variation of Ship Section Parameters on Shear Flow Distribution, Maximum Shear Stresses and Shear Carrying Capacity Due to Longitudinal Vertical Shear Forces", European Shipbuilding, Vol. 18. (Norway-1969), Shama, M. A.,

2- "Effect of Ship Section Scantlings and Transverse Position of Longitudinal Bulkheads on Shear Stress Distribution and Shear Carrying Capacity of Main Hull Girder", Intern. Shipb. Progress, Vol. 16, No. 184, (Holland-1969), Shama, M. A.,

3- "On the Optimization of Shear Carrying Material of Large Tankers", SNAME, J.S.R, March. (USA-1971), Shama, M. A.,

4- " An Investigation into Ship Hull Girder Deflection", Bull. of the Faculty of Engineering, Alexandria University, Vol. XII., (Egypt-1972), Shama, M. A.,

5- "Effective breadth of Face Plates for Fabricated Sections", Shipp. World & Shipbuilders, August, (UK-1972), Shama, M. A.,

6- " Calculation of Sectorial Properties, Shear Centre and Warping Constant of Open Sections", Bull., Of the Faculty of Eng., Alexandria University, Vol. XIII, (Egypt-1974), Shama, M. A.

7- "A simplified Procedure for Calculating Torsion Stresses in Container Ships", J. Research and Consultation Centre, AMTA, (EGYPT-1975), Shama, M. A.

8- "Structural Capability of Bulk Carries under Shear Loading", Bull., Of the Faculty of Engineering, Alexandria University, Vol. XIII, (EGYPT-1975), Also, Shipbuilding Symposium, Rostock University, Sept. (Germany-1975), Shama, M. A.,

9- "Shear Stresses in Bulk Carriers Due to Shear Loading", J.S.R., SNAME, Sept. (USA-1975) Shama, M. A.,

10- "Analysis of Shear Stresses in Bulk Carriers", Computers and Structures, Vol.6. (USA-1976) Shama, M. A.,

11- "Stress Analysis and Design of Fabricated Asymmetrical Sections", Schiffstechnik, Sept., (Germany-1976), Shama, M. A.,

12- "Flexural Warping Stresses in Asymmetrical Sections" PRADS77, Oct., Tokyo, (Japan-1977), Intern. Conf/ on Practical Design in Shipbuilding, Shama, M. A.,

13- "Rationalization of Longitudinal Material of Bulk Carriers, Tehno-Ocean'88, (Jpan-1988), Tokyo, International Symposium, Vol. II, A. F. Omar and M. A. Shama,

14- "Wave Forces on Space Frame Structure", AEJ, April, (Egypt-1992), Sharaki, M., Shama, M. A., and Elwani. M.,

15- "Response of Space Frame Structures Due to Wave Forces", AEJ, Oct., (Egypt-1992). Sharaki, M., Shama, M. A., and Elwani. M. H.

16- "Ultimate Strength and Load carrying Capacity of a Telescopic Crane Boom", AEJ, Vol.41., (Egyopt-2002), Shama, M. A. and Abdel-Nasser, Y.

FACULTY OF ENGINEERING

CALCULATION OF SECTORIAL PROPERTIES, SHEAR CENTRE AND WARPING CONSTANT OF OPEN SECTIONS

BY

M. A. SHAMA*

Reprinted from Bulletin of the Faculty of Engineering
Alexandria University – Vol XIII : 1 — 1974

Printed by M. K. Alexandria Press
1974

CALCULATION OF SECTORIAL PROPERTIES, SHEAR CENTRE AND WARPING CONSTANT OF OPEN SECTIONS

BY

M. A. SHAMA*

Summary

In nearly all problems dealing with the structural mechanics of open thin-walled sections, the warping constant, $J(\omega)$, is encountered. Its calculation is rather lengthy except for some special sections, where symmetry exists.

In this paper, the sectorial coordinates of open thin-walled sections are introduced. These coordinates are used to calculate the position of the shear centre and the warping constant, $J(\omega)$, of open thin-walled sections. Mathematical and numerical methods are both used in the course of calculations.

The methods given are applied to some typical open thin-walled sections which are normally used in ship structures. For these sections, the position of the shear centre and the warping constant are calculated. Open thin-walled sections having an enforced centre of rotation are also considered. The effect on the position of the shear centre and the warping constant of variation of geometry, for two sections, is investigated and the results are illustrated graphically.

The geometrical and flexural properties of an open section are given in the Appendix so as to allow a direct comparison with the sectorial properties.

---

* *Assoc. Prof., Naval Architecture Dept., Faculty of Eng., Alexandria University, Egypt.*

1. Introduction

Thin-walled sections are widely used in aircraft and ship structures and are becoming very attractive to structural engineers. The economy achieved through the reduced weight/strength ratio makes thin-walled sections very desirable (1).

In the structural mechanics of thin-sections, shear, torsion (2) stability (3) and warping (4) problems become rather significant. The solution of the torsion equation requires the knowledge of the warping constant $J(\omega)$. In structures subjected to torsional buckling, the position of the shear centre and the warping constant, $J(\omega)$, must be determined before any solution could be obtained. The calculation of the shear and flexural warping stresses of thin-walled members are also based on the position of the shear centre and the warping constant $J(\omega)$.

In this paper, the position of the shear centre and the warping constant of thin-walled sections are both determined using the sectorial properties (5). Numerical and direct integration are both used in the course of calculations. Few examples are given to illustrate the simplicity of using the sectorial properties of thin-walled sections for calculating the warping constant. In this respect, symmetrical and asymmetrical sections are considered together with sections having an enforced centre of rotation.

2 Sectorial Properties of Thin-Walled Sections.

In addition to the geometrical and flexural properties of sections, i.e. A, S_x, S_y, I_x, I_y and I_{xy}, there are additional unique characteristics for thin-walled sections which are called «Sectorial

Properties» (5). These properties are also associated with area, moment of area and moment of inertia.

The sectorial properties of a thin-walled open section, see fig. (1), having an arbitrary pole P and an arbitrary starting point 0, are as follows (5) :

- Sectorial area $= \omega = \int_{s_2}^{s_1} r \, ds \quad cm^2 \qquad (2.1)$

where r = the perpendicular distance from the pole P to the tangent at the point under consideration; see fig. (1).

s_1 and s_2 represent end points on the contour of section.

- Sectorial static moment $= S(\omega) = \int_A \omega \, dA \quad cm^4 \qquad (2.2)$

- Sectorial linear moments,

$$S(\omega)_x = \int_A \omega \, y \, dA \quad cm^5 \qquad (2.3)$$

$$S(\omega)_y = \int_A \omega \, x \, dA \quad cm^5 \qquad (2.4)$$

- Sectorial moment of inertia $= J(\omega) = \int_A \omega^2 \, dA \quad cm^6 \qquad (2.5)$

For a thin-walled section having «n» elements, each having a uniform thickness «t» and a constant «r»,

$$J(\omega) = \frac{1}{3} \sum_{i=1}^{n} r_i^2 \cdot t_i \cdot (s_i^3)_{s_1}^{s_2} \qquad (2.5a)$$

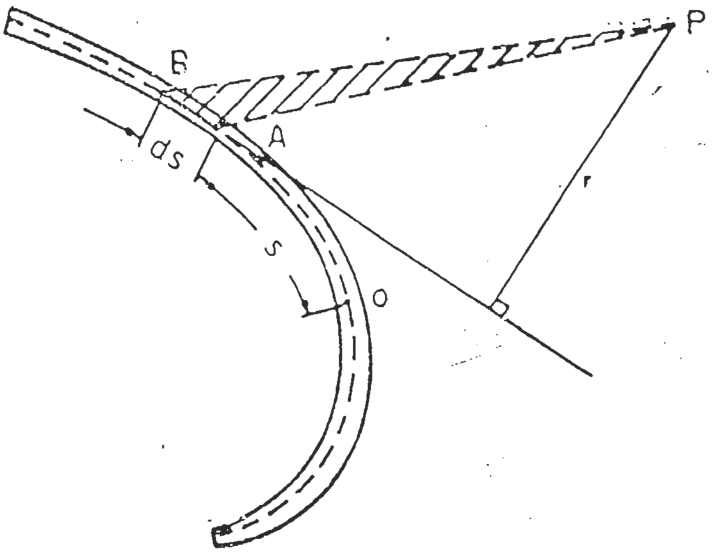


Fig.(1), Sectorial Coordinates

It is to be noted here that once the sectorial area diagram is determined, the sectorial characteristics of the section could be easily calculated by either direct or numerical integration. In the majority of cases, the integral of the product of two functions is required to be evaluated. A simplified method for evaluating this integral is given in Appendix (1).

3. Principal Sectorial Properties of Thin-Walled Sections.

In the same way as for the principal centroidal axes, there is also the principal sectorial coordinates. The pole of the latter system is the «principal pole» or the shear centre of the section. The principal origin is defined by the «principal radius». The principal sectorial moment of inertia J (ω) is calculated using the principal sectorial coordinates.

The «principal pole» and the «principal radius» are determined from the following conditions:

$$S(\omega) = \int_A \omega \, . \, dA = 0 \qquad (3.1)$$

$$S(\omega)_X = \int_A \omega \, . Y . dA = 0 \qquad (3.2)$$

$$S(\omega)_Y = \int_A \omega \, . X . dA = 0 \qquad (3.3)$$

Condition (3.1) gives the direction of the principal radius i.e. the location of the «principal origin» whereas conditions (3.2) and (3.3) give the location of the principal pole, i.e. the shear centre. From these conditions, it is evident that the sectorial linear moments with respect to the principal centroidal axes and a pole coincident with the shear centre are zero. The origin of ω is of no importance because if it is shifted, the sectorial area diagram is changed by a constant which does not have any effect on conditions (3:2) and (3.3).

Therefore, in order to determine the location of the shear centre, a diagram of sectorial area ω' for an arbitrary polo P' is drawn. The location of the shear centre is given by :

$$e_X = -\int_A \omega' . Y \; dA / I_X \qquad (3.4)$$

$$e_Y = \int_A \omega' . X . dA / I_Y \qquad (3.5)$$

where X and Y are the principal centroidal axes of the section, I_X and I_Y are the principal moments of intertia about the X and Y axes respectively,

e_X and e_Y are the coordinates of the shear centre with respect to the assumed pole P'.

However, if the centroidal axes x and y are not the principal centroidal axes X, Y, the coordinates of the shear centre with respect to the assumed pole P' are given by ;

$$e_x = -\left(I_y . \int_A \omega' . y . dA - I_{xy} . \int_A \omega' . x . dA\right) / \left(I_x I_y - I_{xy}^2\right) \qquad (3.6)$$

$$e_y = \left(I_x . \int_A \omega' . x . dA - I_{xy} . \int_A \omega' . y . dA\right) / \left(I_x I_y - I_{xy}^2\right) \qquad (3.7)$$

It is to be noted here that once the sectorial area diagram is determined, the sectorial characteristics of the section could be easily calculated by either direct or numerical integration. In the majority of cases, the integral of the product of two functions is required to be evaluated. A simplified method for evaluating this integral is given in Appendix (1).

3. Principal Sectorial Properties of Thin-Walled Sections.

In the same way as for the principal centroidal axes, there is also the principal sectorial coordinates. The pole of the latter system is the «principal pole» or the shear centre of the section. The principal origin is defined by the «principal radius». The principal sectorial moment of inertia J (ω) is calculated using the principal sectorial coordinates.

The «principal pole» and the «principal radius» are determined from the following conditions:

$$S(\omega) = \int_A \omega \,.\, dA = 0 \qquad (3.1)$$

$$S(\omega)_X = \int_A \omega \,.\, Y \,.\, dA = 0 \qquad (3.2)$$

$$S(\omega)_Y = \int_A \omega \,.\, X \,.\, dA = 0 \qquad (3.3)$$

Condition (3.1) gives the direction of the principal radius i.e. the location of the «principal origin» whereas conditions (3.2) and (3.3) give the location of the principal pole, i.e. the shear centre. From these conditions, it is evident that the sectorial linear moments with respect to the principal centroidal axes and a pole coincident with the shear centre are zero. The origin of ω is of no importance because if it is shifted, the sectorial area diagram is changed by a constant which does not have any effect on conditions (3.2) and (3.3).

Therefore, in order to determine the location of the shear centre, a diagram of sectorial area ω' for an arbitrary pole P' is drawn. The location of the shear centre is given by :

$$e_X = -\int_A \omega' . Y \; dA / I_X \qquad (3.4)$$

$$e_Y = \int_A \omega' . X . dA / I_Y \qquad (3.5)$$

where X and Y are the principal centroidal axes of the section, I_X and I_Y are the principal moments of intertia about the X and Y axes respectively,

e_X and e_Y are the coordinates of the shear centre with respect to the assumed pole P'.

However, if the centroidal axes x and y are not the principal centroidal axes X, Y, the coordinates of the shear centre with respect to the assumed pole P' are given by ;

$$e_x = -\left(I_y . \int_A \omega' . y . dA - I_{xy} . \int_A \omega' . x . dA\right) / \left(I_x I_y - I_{xy}^2\right) \qquad (3.6)$$

$$e_y = \left(I_x . \int_A \omega' . x . dA - I_{xy} . \int_A \omega' . y . dA\right) / \left(I_x I_y - I_{xy}^2\right) \qquad (3.7)$$

where I_x, I_y and I_{xy} are the moments and product of inertia about the centroidal axes x,y.

In order to determine the principal sectorial area diagram, the shear centre is used as the principal pole and an arbitrary origin O'' is assumed. The sectorial area diagrm ω'' is calculated. The principal sectorial area at any point is given by :

$$\omega = \omega'' + \omega_c \qquad (3.8)$$

where ω_c is a constant given by :

$$\omega_c = -\int_A \omega'' . dA / A_T \qquad (3.9)$$

The calculation of the principal sectorial area diagram could be summarised as follows :

i) The flexural properties of the section about the principal centroidal axes X, Y or any convenient centroidal axes x,y are determined, i.e. I_X, I_Y or I_x, I_y and I_{xy} in addition to the sectional, area A, as given in Appendix (2).

ii) Choose an arbitrary pole P' and an arbitrary origin O', and then calculate the sectorial area diagram ω'.

iii) Calculate the coordinates of the shear centre relative to P' using either expressions (3.4) and (3.5) or (3.6) and (3.7), depending on whether the axes X,Y or x,y are used.

iv) Using the calculated shear centre as the principal pole and assuming any arbitrary origin O'', the sectorial area diagram ω'' is calculated.

v) The correcting term ω_c is then calculated using expression (3.9).

vi) The principal sectorial area diagram ω is then calculated using expression (3.8).

4. Applications to Some Typical Sections

The position of the shear centre and the sectorial moment of inertia, $J(\omega)$, are calculated for the following sections, using the principal sectorial area diagram :

a — I — Section.

Due to symmetry, see fig. (2), the shear centre coincides with the centroid of the section, i.e.

$$e_X = e_Y = 0$$

$$J(\omega) = \int_A \omega^2 . dA = 4 t_f . \int_0^{b/2} \left(\frac{d.s}{2}\right)^2 . ds$$

$$= I_Y . \frac{d^2}{4} \qquad (4.1)$$

$J(\omega)$ could be also calculated using the method given in Appendix (1) as follows :

$$J(\omega) = 4 . \left(\frac{1}{2} \quad \frac{d.b}{4} . t_f . \frac{b}{2} . \frac{2}{3} \quad \frac{d.b}{4}\right) = I_Y . \frac{d^2}{4}$$

b — Channel section.

Due to symmetry about the X—axis, see fig. (3), we have :

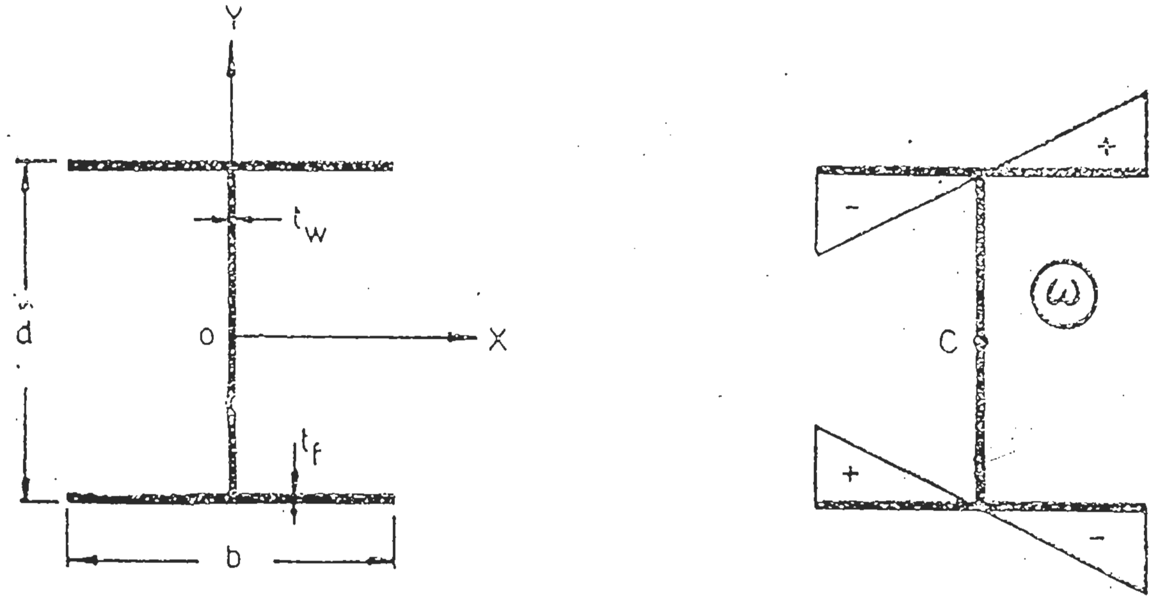


Fig.(2). Sectorial Area Diagram for I-Section.

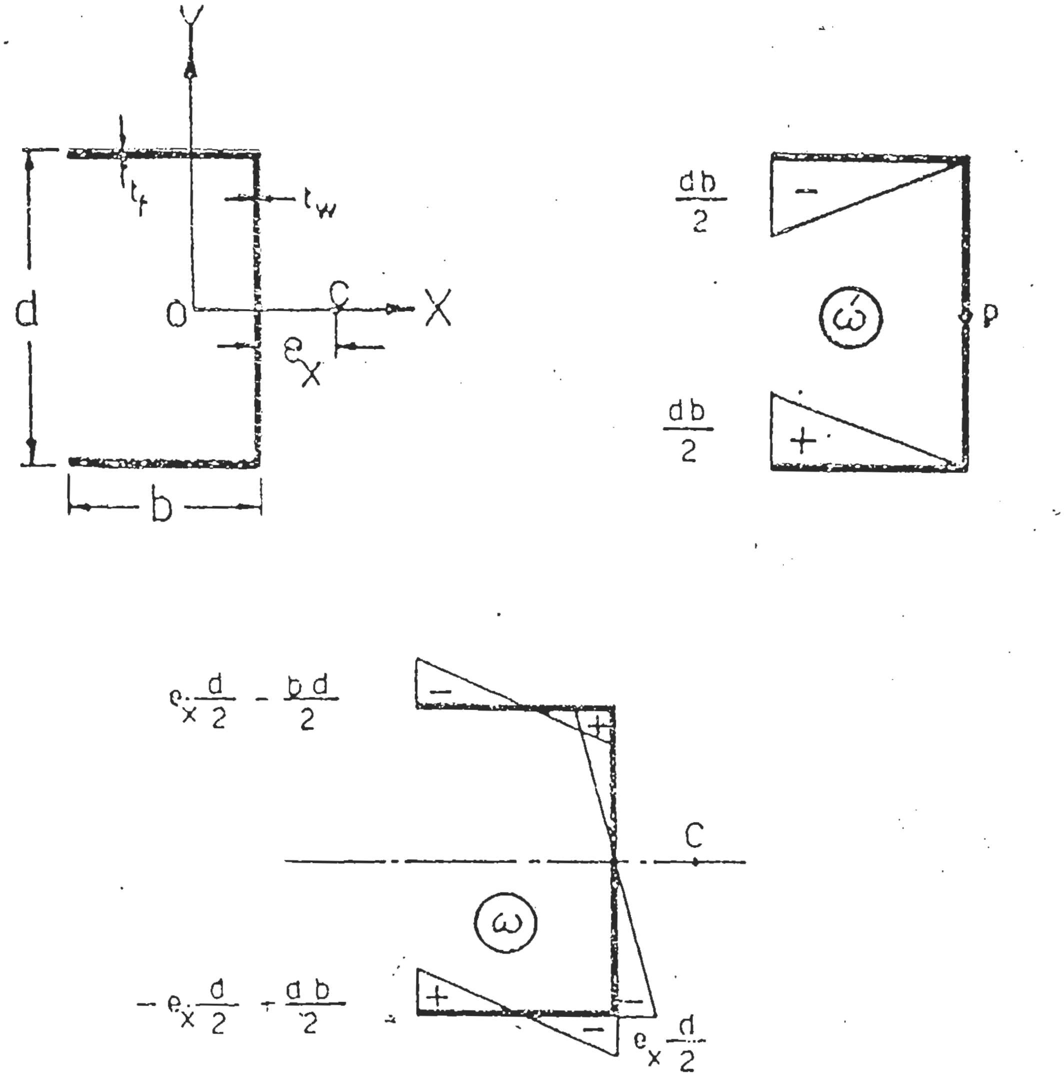


Fig.(3), Sectorial Area Diagram for a Channel

$$e_Y = 0$$

The shear centre is, therefore, located along the X– axis at a distance given by :

$$e_X = - \int_A \omega \, Y . dA \, / \, I_X$$

$$I_X = d^2 . (d \, t_w / 12 + b . t_f / 2)$$

$$\int_A \omega' . Y . dA = t_f \, \frac{b.d}{4} \int_c^b (-s) . ds - t_f . \frac{b.d}{4} . \int_0^b s \, ds$$

$$= - t_f . b^2 . d^2 / 4$$

Hence, $e_X = b / (A_w / 3 A_f + 2)$

$$J(\omega) = \int_A \omega^2 . dA$$

$$= 2 \, t_w . \int_0^{d/2} (e_X . s)^2 . ds + 2 t_f . \int_0^b (e_X . \frac{d}{2} - \frac{d.s}{2})^2 . ds$$

$$= I_X . e_X^2 + t_f . b^2 . d^2 . (b - 3 \, e_X) / 6$$

The variation of e_X and $J(\omega)$ with geometry are shown in figs. (4) and (5) respectively.

c – Asymmetrical I - section.

Due to symmetry about the Y – axis, see fig. (6), we have :

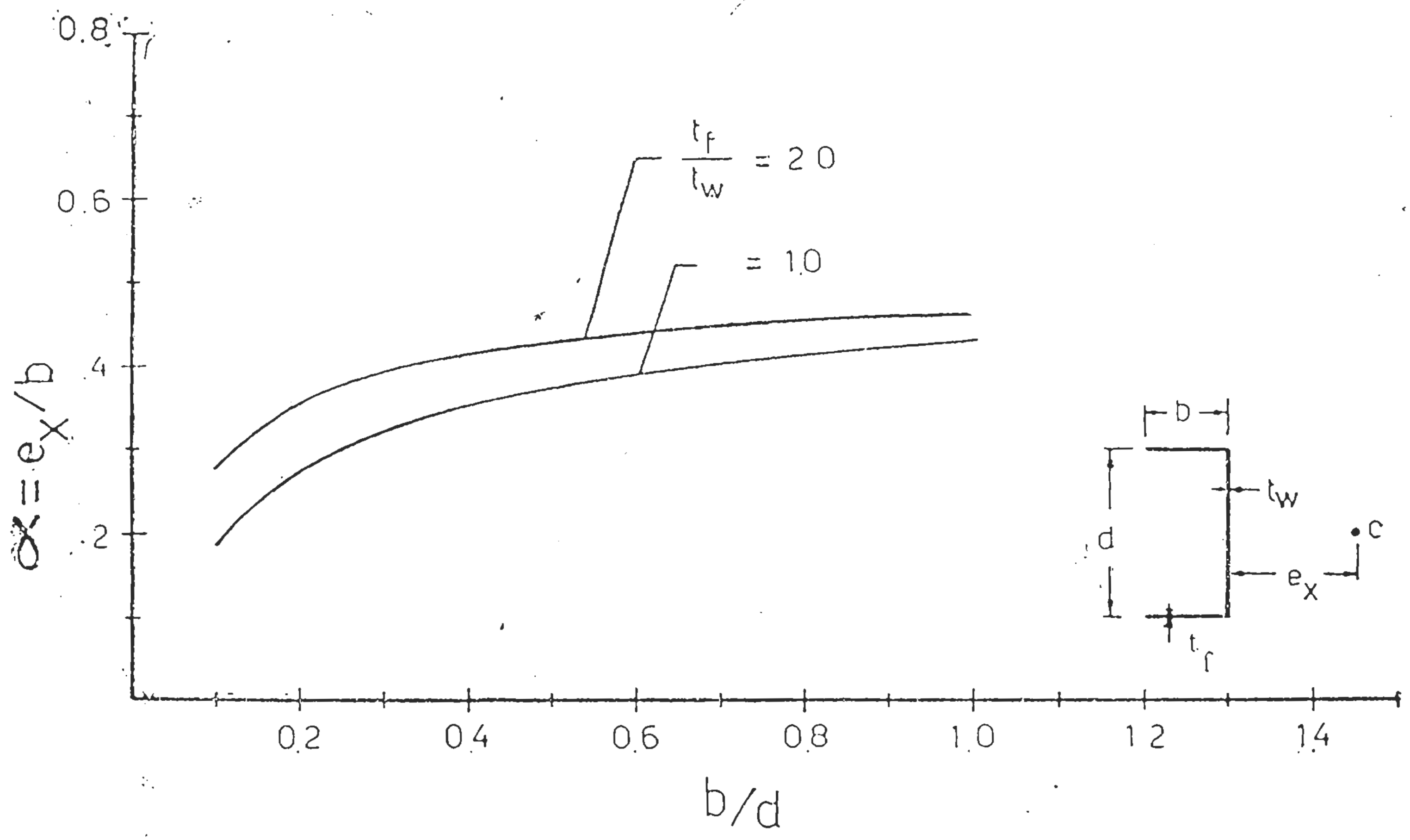


Fig.(4). Variation of e_x with the Geometry of Channel

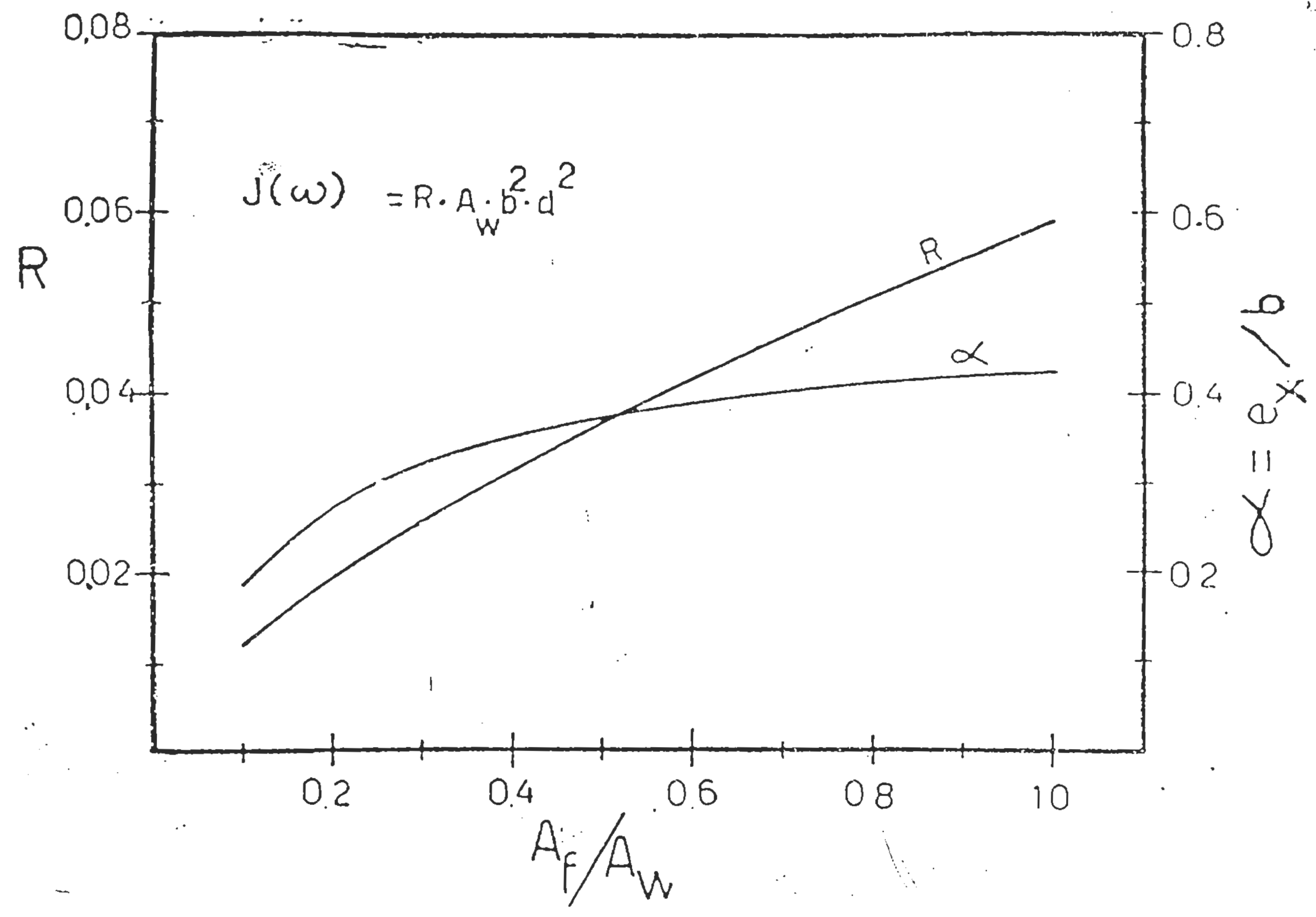


Fig.(5). Variation of $J(\omega)$ with the Geometry of Channel.

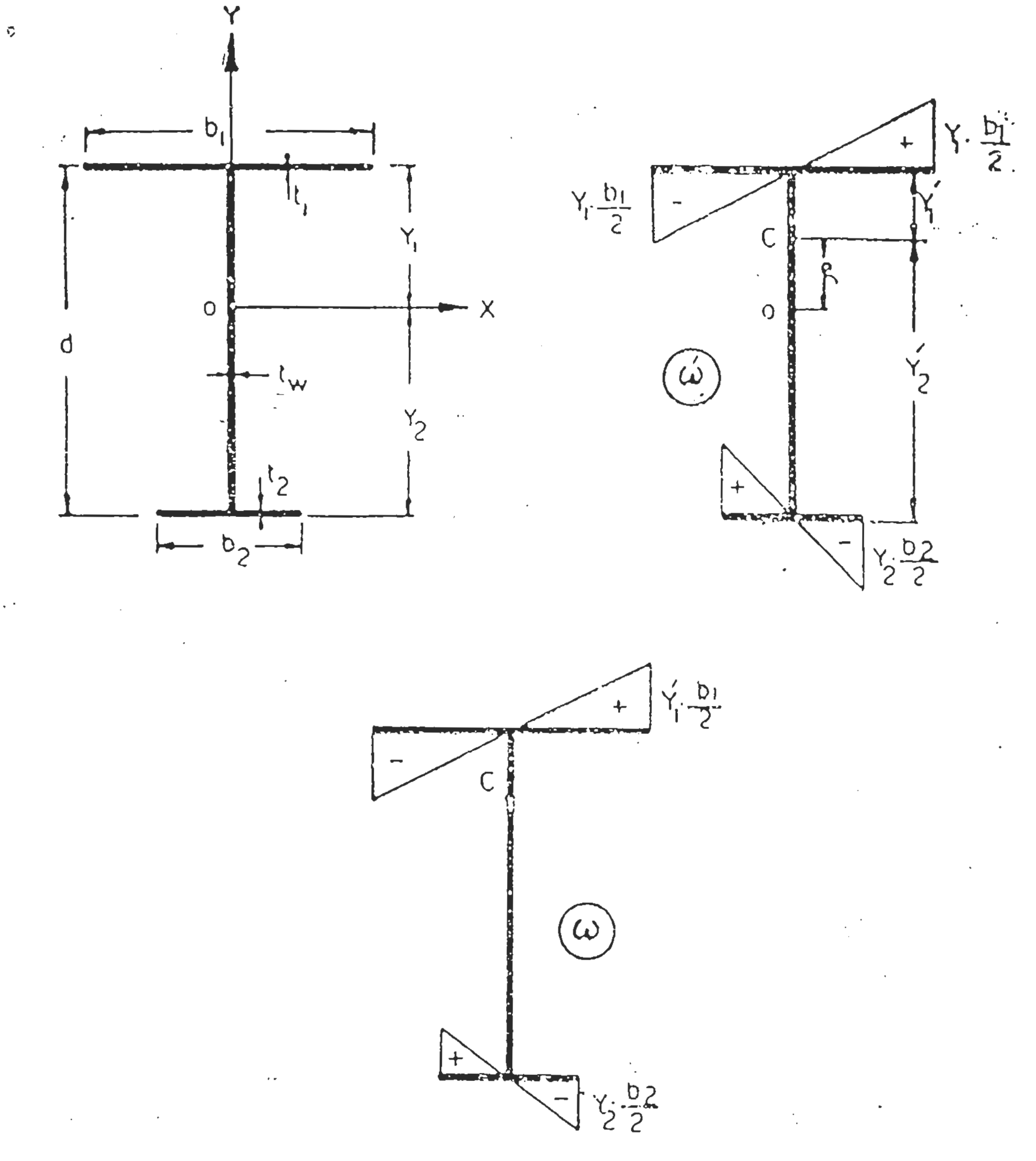


Fig.(6). Sectorial Area Diagram for an Asymmetrical I-Section.

$$e_X \doteq 0$$

$$e_Y = \int_A \omega' \; X \; dA \, / \, I_Y$$

$$I_Y = t_1 . b_1^3 / 12 + t_2 \, b_2^3 / 12 = I_1 + I_2$$

where I_1 and I_2 are the moments of inertia of the top and bottom flanges respectively.

$$\int_A \omega' . X . dA = 2t_1 . Y_1 . \int_0^{b_1/2} s^2 . ds + 2t_2 . Y_2 . \int_0^{b_2/2} s^2 . ds$$

$$= I_1 . Y_1 \quad I_2 . Y_2$$

Hence, $e_Y = (I_1 . Y_1 + I_2 . Y_2) / I_Y$

where $Y_1 = d . (2 A_1 + A_W) / 2 A_T$

$Y_2 = d . (2 A_2 + A_W) / 2 A_T$

and $Y_1' = Y_1 - e_Y \doteq d . I_2 / I_Y$

$Y_2' = d - Y_1' = d . I_1 / I_Y$

$$J(\omega) = \int_A \omega^2 \, dA$$

$$= 2t_1 . \int_0^{b_1/2} (Y_1' \; s)^2 . ds + 2t_2 . \int_0^{b_2/2} (Y_2' . s)^2 . ds$$

$$= I_1 . I_2 . d^2 / I_Y$$

for a symmetrical I - section

$$J(\omega) = I_Y \, d^2 / 4$$

The variation of $J(\omega)$ with geometry of section is shown in fig. (7).

d — A T-section with an enforced axis of rotation.

In this case, the enforced axis of rotation replaces the shear centre when calculating the principal sectorial area diagram, see fig. (8). The sectorial moment of inertia is, therefore, given by :

$$J(\omega) = \int_A \omega^2 \, dA = 2 t_f \int_0^{-b/2} (d\, s)^2 \, ds = I_Y \, d^2$$

e - An asymmetrical section with an enforced axis of rotation

The sectorial moment of inertia $J(\omega)$ for the asymmetrical section shown in fig. (8) is given by :

$$J(\omega) = \int_A \omega^2 \, dA$$

where,

$$\omega = \omega' + \omega_c$$

$$\omega_c = -\frac{1}{A_T} \int_A \omega' \, dA = -\frac{b.d.A_f}{2 A_T}$$

Hence,

$$J(\omega) = \int_A (\omega' + \omega_c)^2 \, dA$$

$$= I_Y . d^2 - \frac{b^2 \, d^2 \, A_f^2}{4 A_T}$$

where,

$$A_T = d . t_w + b . t_f = A_W + A_f$$

$$I_Y = A_f . b^2 / 3$$

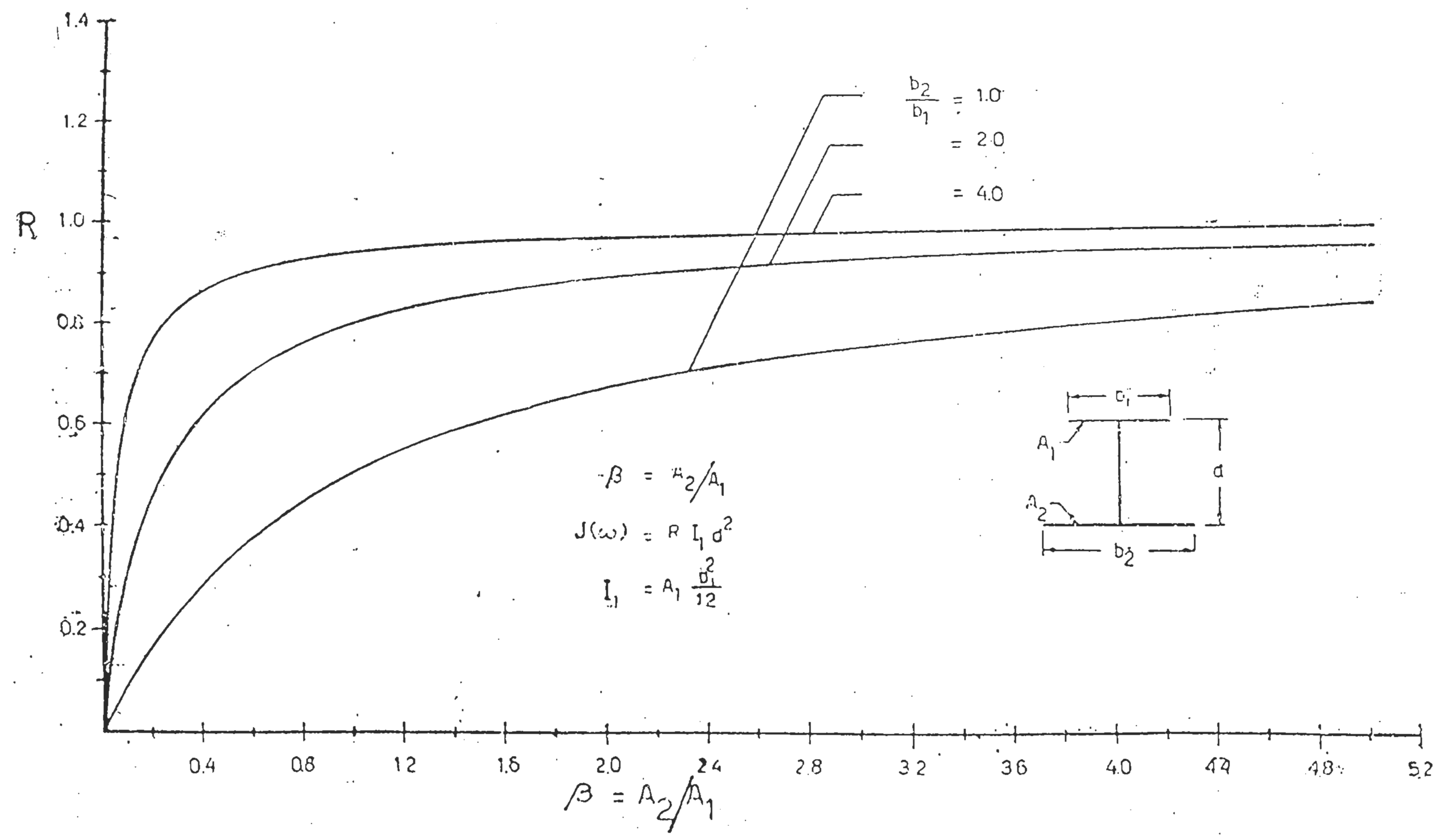


Fig.(7). Variation of R with Geometry of Section.

f — An asymmetrical section with an enforced axis of rotation at point C on the opposite side of the face plate.

This is a typical case of an asymmetrical stiffener subjected to lateral loading. The enforced axis of rotation is at a distance e from the web of the stiffener and should be within the plating as the latter cannot deform in its own plane, see fig.(9).

The coordinates of the enforced axis of rotation, in the plane of the section, are $(-p-e)$; $(-y_2)$. The sectorial coordinates of the section are given by :

$$\omega = \omega' + \omega_c$$

$$\text{where, } \omega_c = -\frac{1}{A_T} \int_A \omega' \cdot dA = -\frac{1}{A_T} \sum_A \omega' \, dA$$

A method for calculating ω_c is given in Table (1), using numerical integration. Thus,

$$\omega_c = -\frac{d}{A_T} \cdot \left[e \, A_w + (2e - b) \cdot A_f \right]$$

Hence, the sectorial moment of inertia, $J(\omega)$, is given by :

$$J(\omega) = \int_A \omega^2 . dA = \int_A (\omega' + \omega_c)^2 . dA = \sum_A \omega^2 \, dA$$

Table (2) shows a method for calculating $J(\omega)$, using numerical integration. $J(\omega)$ is, therefore, given by :

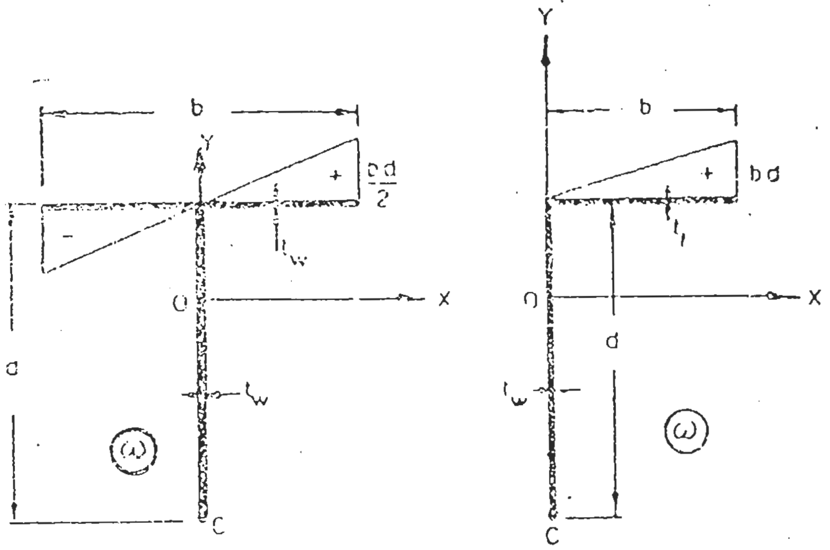


Fig.(8). ω Diagram for Enforced Centre of Rotation.

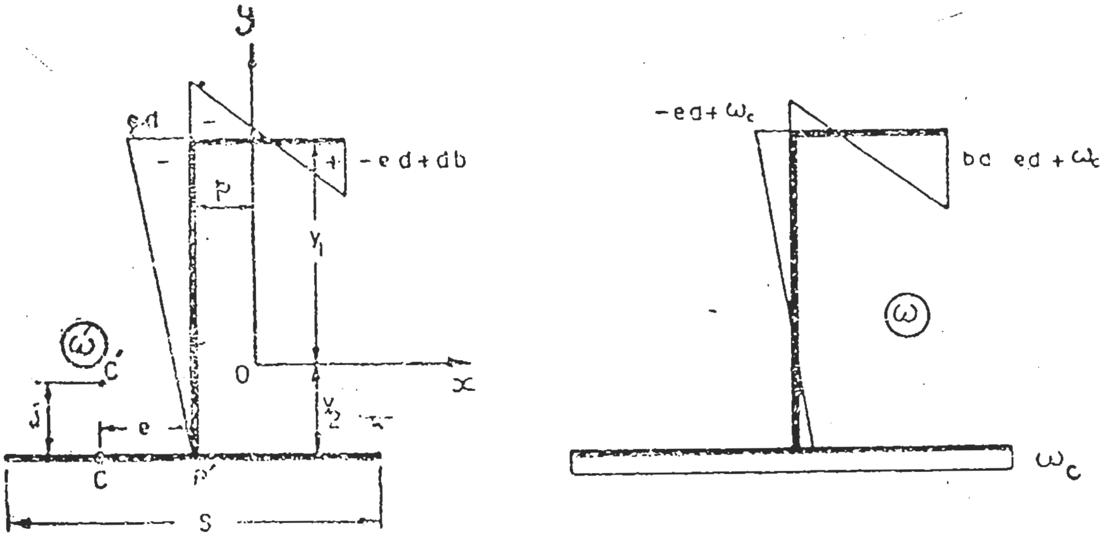


Fig.(9). Enforced Centre of Rotation Outside the Section.

$$J(\omega) = d^2 \cdot \left[A_w \cdot \frac{e^2}{3} + A_f \cdot \left(e^2 + \frac{b^2}{3} - e \cdot b \right) \right] - \frac{d^2}{4A_T} \left[e\, A_w + (2e - b) \cdot A_f \right]^2$$

where, $A_T = S \cdot t_p + d \cdot t_w + b \cdot t_f$

For an isolated member, having this type of section, the shear centre C', see fig. (9), is located at a distance e from the web and j from the outer plating, as given in reference (6), where,

$$e = \frac{A_p \cdot S^2 \cdot I_{yx} \cdot d}{12 (I_x \cdot I_y - I_{xy}^2)}$$

$$j = \frac{\left[I_x \cdot (I_y - \frac{A_p \cdot S^2}{12}) - I_{xy}^2 \right] d}{I_x \cdot I_y - I_{xy}^2}$$

For an enforced axis of rotation lying within the outer plating, the distance $j = 0$.

5. Shear and Flexural Warping Stresses.

The sectorial properties of an open thin-walled section are used to calculate the shear and flexural warping stresses as follows :

$$\tau (\omega) = \frac{T(\omega)}{t \cdot J(\omega)} \cdot S(\omega) \qquad (5.1)$$

$$\sigma (\omega) = \frac{B \cdot \omega}{J(\omega)} \qquad (5.2)$$

where, $T(\omega)$ = warping torsional moment;

$\sigma(\omega)$ = flexural warping stress

$\tau(\omega)$ = shear stress due to flexural warping

B = a mathematical function introduced by Vlasov (5) and is called «Bimoment»

The calculation of shear and flexural warping stresses is not the purpose of this paper but will be considered in detail in a future publication.

6. Concluding Remarks.

In nearly all problems dealing with the structural mechanics of open thin-walled sections, the warping constant $J(\omega)$ is encountered. Its calculation is rather lengthy except for some special sections, where the symmetry simplifies the computations.

The calculation of $J(\omega)$, as well as the position of the shear centre, is greatly simplified when the sectorial coordinates of the section are made use of. The sectorial properties of a section could be determined for both symmetrical and asymmetrical sections, even when the section is constrained to rotate about an enforced axis of rotation.

In the course of calculating the sectorial properties, position of shear centre and the warping constant of an open thin-walled section, either direct or numerical integration could be used. A method is given to simplify the use of numerical integration when one of the functions to be integrated is varying linearly over the contour of the section.

The sectorial coordinates are used to calculate the warping constant for some typical open thin-walled sections normally used in ship structures. The effect of section parameters on the location of the shear centre. and J(ω), for a channel and an I–section, is investigated and the results are illustrated in a graphical form. The presence of an enforced axis of rotation, for both symmetrical and asymmetrical sections, has also been considered.

7. References :

1. Shanley, F. R., «Weight – Strength Analysis of Aircraft Structures». Dover Publication, 1960.
2. Timoshenko, S., «Theory of Bending, Torsion and Buckling of Thin-Walled Members of Open Cross-Section». Journal of the Franklin Institute, Philadelphia, March, April and May 1945.
3. Bleich, F., «Buckling Strength of Metal Structures»; Mc-Graw-Hill Co, 1952.
4. Shama, M. A., «Effective Breadth of Face Plates For Fabricated Sections», Shipping World and Shipbuilder, August, 1972.
5. Vlasov, V.Z. «Thin–Walled Elastic Beams»; English Translation, National Science Foundation; Washington D.C.; London Old–bourne Press, 1961.
6. Yuille, I.M., «Application of the Theory of Elastic Bending to the Structural Members of Ships», RINA April. 1960.
7. Oden, J. T., «Mechanics of Elastic Structures», McGraw–Hill Co. 1967.
8. Feodosyev, V., «Strength of Materials», MIR Publishers, Moscow, 1968.

8. Acknowledgements :

The author wishes to thank prof. M.H. Said Ahmed for his valuable comments and Eng. O.M. Khattab for tracing the drawings.

Appendix (1)

The Integral of the Product of Two Functions.

In order to perform the following integral.

$$I = \int_0^{\ell} x \cdot y \cdot ds$$

Where x and y are both functions of s, it is necessary that either x or y, or both, are linear functions of s.

If y is the linear function of s. see fig. (10), then :

$$y = a + b \cdot s$$

and

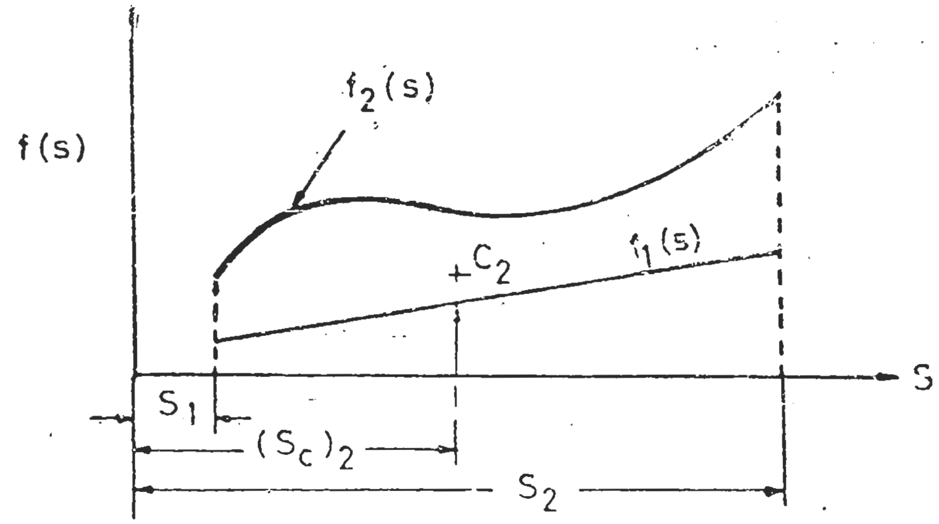
$$I = \int_0^S a \cdot x \cdot ds + \int_0^S b \cdot s \cdot x \cdot ds$$

$$= a \cdot \int_0^S x \cdot ds + b \cdot \int_0^S x \cdot s \cdot ds$$

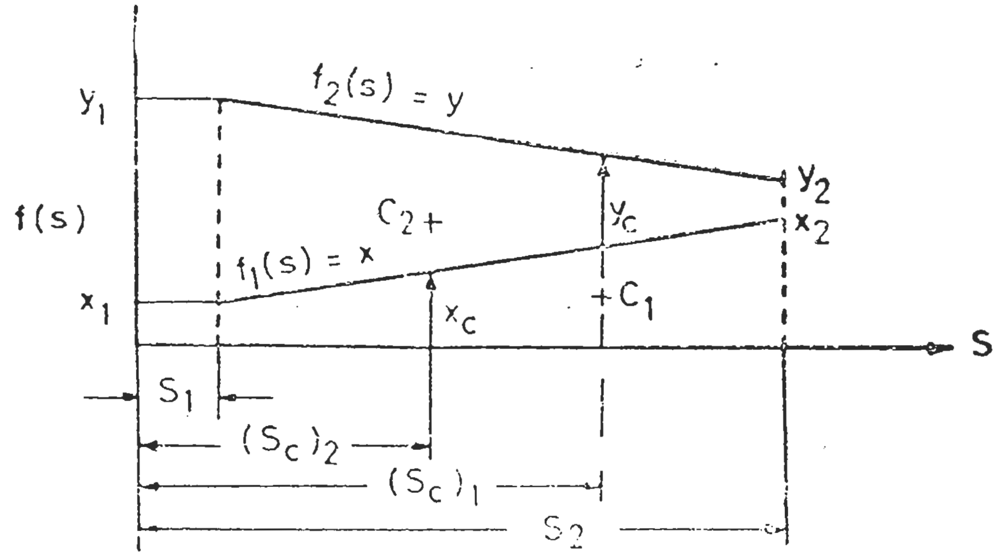
$$= a \cdot A_x + b \cdot A_x \cdot S_c$$

$$= (a + b \cdot S_c) \cdot A_x$$

$$= y_c \cdot A_x$$



a- $$I = \int_{s_1}^{s_2} f_1(s) \cdot f_2(s) \cdot ds = A_2 \cdot f_1(s_c)_2$$



b- $$I = \int_{s_1}^{s_2} f_1(s) \cdot f_2(s) \cdot ds$$

$$= A_x \cdot y_c = A_y \cdot x_c$$

FIG (10). THE INTEGRAL OF THE PRODUCT OF TWO FUNCTIONS

If x and y are both linear functions of s, the integral I is given by :

$$I = A_x \cdot y_c = A_y \cdot x_c ,$$

Where; y_c = value of y at the centroid of x, i.e. at s_c of x,

x_c = value of x at the centroid of y; i.e. at s_c of y.

It should be noted that the sectorial properties are normally given in terms of the end points of each member of the thin-walled section. The calculation of the sectorial properties of a section could be, therefore, based on a simplified expression, as given by :

$$I = \frac{S}{6} \left[y_1 (2 x_1 + x_2) + y_2 (x_1 + 2 x_2) \right]$$

where: x_1, x_2, y_1 and y_2 are the end ordinates of the two functions; see fig. (10).

If, however, the two functions are expressed in an analytic form, direct integration could be used.

Appendix (2)

Geometrical and Flexural properties of a Section.

These properties are given in the majority of text books on structural mechanics (7) and (8) and are given here only for the sake of comparison with the corresponding sectorial properties.

For any arbitrary section having centroidal axes x,y, the geometrical and flexural properties are given by :

i) Sectional area, A

$$A = \int_A y \, . \, dx = \int_A x \, . \, dy \qquad (A1)$$

ii) First moment of area about the x and y axes respectively, i. e., S_x and S_y.

$$S_x = \int y \, . \, dA \quad , \quad S_y = \int x \, . \, dA \qquad (A2)$$

Since x and y are centroidal axes,

$$S_x = \int_A y \, . \, dA = 0 \quad , \quad S_y = \int_A x \, . \, dA = 0 \qquad (A3)$$

These two equations determine the location of the centroid of the section.

iii) Second moment of area about the x and y axes.

$$I_x = \int_A y^2 \, . \, dA \quad , \quad I_y = \int_A x^2 \, . \, dA \quad , \quad I_{xy} = \int_A x \, . \, y \, . \, dA \qquad (A4)$$

These geometrical and flexural properties are used to calculate the normal and flexural stresses in a uniform member. These stresses are given by :

a. normal stress, σ_n

$$\sigma_n = P / A \qquad (A5)$$

provided that St. Venant's principle has been observed.

b. shear stress; τ_{xy}

$$\tau_{xy} = -\frac{S_x . I_y - S_y . I_{xy}}{t (I_x . I_y - I_{xy}^2)} . F_y + \frac{S_y . I_x - S_x . I_{xy}}{t (I_x . I_y - I_{xy}^2)} . F_x \qquad (A6)$$

c. flexural stress, σ_f

$$\sigma_f = -\frac{I_y . y - I_{xy} . x}{I_x . I_y - I_{xy}^2} . M_x + \frac{I_x . x - I_{xy} . y}{I_x . I_y - I_{xy}^2} . M_y \qquad (A7)$$

If the coordinate axes X, Y are the principal centroidal axes, the product of inertia given in (A4) must be zero, i.e ,

$$I_{XY} = \iint_A X . Y . dA = 0 \qquad (A8)$$

Equation (A8) determines the orientation of the principal centroidal axes. Substituting (A8) into (A6) and (A7), we get :

$$\tau_{XY} = S_X \, F_Y / t \, I_X \cdot S_Y . F_X / t . I_Y \qquad (A9)$$

$$\sigma_f = M_X . Y / I_X \cdot M_Y . X / I_Y \qquad (A10)$$

TABLE (1)

Calculation of the sectorial coordinates for an asymmetrical section having an enforced axis of rotation

member	ω'_1	ω'_2	ω'_m	A	$A\ \omega'_m$
1 - 2	0	0	0	$t_p \cdot \frac{S}{2}$	0
2 - 3	0	0	0	$t_p \cdot \frac{S}{2}$	0
2 - 4	0	$-ed$	$-ed/2$	$d\,t_w$	$-d^2 e\,t_w/2$
4 - 5	$-ed$	$-ed+bd$	$-ed+bd$	$b\,t_f$	$b\,t_f\left(\frac{bd}{2} - ed\right)$

$$A_I = \sum_A \omega'\, dA$$

TABLE (2)

Calculation of $J(\omega)$ for an asymmetrical section having an enforced axis of rotation

member	ω_1^2	ω_2^2	ω_m^2	$\omega^2\ dA$
1 - 2	ω_c^2	ω_c^2	ω_c^2	$A_p\ \omega_c^2/2$
2 - 3	ω_c^2	ω_c^2	ω_c^2	$A_p\ \omega_c^2/2$
2 - 4	ω_c^2	$(\omega_c - ed)^2$	$(\omega_c - \frac{ed}{2})^2$	$(\omega_1^2 + 4\omega_m^2 + \omega_2^2)\ \frac{A_w}{6}$
4 - 5	$(\omega_c - ed)^2$	$(\omega_c - ed + bd)^2$	$(\omega_c - ed + \frac{bd}{2})^2$	$(\omega_1^2 + 4\omega_m^2 + \omega_2^2)\ \frac{A_f}{6}$

$$J(\omega) = \sum_A \omega^2\, dA$$

List of Notation.

A = sectional area

A_f = sectional area of face plate.

A_p = sectional area of outer plating.

A_T = total sectional area.

A_w = sectional area of web.

b = width of face plate.

B = Bimoment.

c = constant.

d = depth of section.

e = distance of shear centre from outer plating.

e_x, e_y = coordinates of the shear centre relative to the x and y axes.

e_X, e_Y = coordinates of the shear centre relative to the X and Y axes.

F_x, F_y = shear forces in the direction of the x and y - axes,

I = integral of the product of two functions.

I_x, I_y = second moment of area about the x and y axes, respectively.

I_{xy} = product of inertia about the x and y axes.

I_X, I_Y = second moment of area about the X and Y axes, respectively.

j = distance of the shear centre from the web plating.

$J(\omega)$ = warping constant of a section.

M_x, M_y = bending moments about the x and y axes, respectively.

P = normal force.

r = perpendicular distance from the pole to the tangent at any point on the contour of the section.

S = effective breadth of outer plating.

S_x, S_y = moments of area about the x and y axes, respectively.

$S(\omega)$ = sectorial static moment.

$S(\omega)_x$, $S(\omega)_y$ = sectorial linear moments about the x and y axes, respectively.

ds = elementary length on the contour of a section.

t = thickness.

t_f = thickness of face plate.

t_p = thickness of outer plating.

t_w = thickness of web plate.

x, y = centroidal axes; (in Appendix (1) they represent two functions of s).

X, Y = principal centroidal axes.

x_c = ordinate of the function x at the centroid of the function y.

y_c = ordinate of the function y at the centroid of the function x

ω = sectorial area.

ω_c = correcting sectorial area.

ω_m = mean sectorial area

σ_n = normal stress.

σ_f = flexural stress.

$\sigma(\omega)$ = flexural warping stress.

τ_{xy} = shear stress.

$\tau(\omega)$ = warping shear stress.